# Replacement

of Section 12.4 in

Klaus Krippendorff. *Content Analysis; An Introduction to Its Methodology*. 3rd Edition
Thousand Oaks, CA: Sage Publications, 2013

To be introduced into its **2nd printing**

Originally posted 2014.4.12, revised 2015.9.23

kkrippendorff@asc.upenn.edu

## Agreement when Unitizing and Coding Finite Continua 12.4

Most content analyses sample large and coherent texts: books, speeches, narratives, TV shows, video recordings, transcripts of interviews, streams of e-mail messages, or websites. These essentially are or can be conceived of as textual sequences, as finite continua. While one tends to have no trouble reading such continua and extracting meaningful information from them, without the ability to establish the reliability of identifying relevant sections in such continua, the reliabilities of subsequent coding, or analyzing such data remain on uncertain ground. This section seeks to extend reliability considerations to the processes of unitizing given continua.

Tabulating, comparing, or counting natural units seems unproblematic. Chapter 8 addresses various easily recognizable units. When such units exist, attention can shift to the reliability of coding. However, uniformly applicable units are not always available. In such situations, content analysts tend to define standard units to suit their analytical purposes. Pages of books, 30-second intervals of conversations, scenes in film, or the first 100 words of articles are easily separable and countable, give the researcher the impression of objectivity, but can also create meaningless data. For example, topics can hardly be distinguished by book pages. Analyzing talk shows on radio in 30-second intervals may cut question-answer sequences into uninterpretable segments, and distinguishing camera shots in film may make sense to film editors but rarely corresponds to how viewers conceive of what they see. Relying on units that do not fit the nature of the phenomena of interest is likely to introduce uncertainties into the data without the analysts' knowing whether, why, and their extent.

On the other extreme, qualitative scholars have advocated relying on units of texts that emerge in the process of reading. For understanding larger narratives, it is quite natural to select key quotes from speeches, clip informative paragraphs from newspapers, segment conversations by turns taken, or highlight and code analytically relevant passages of text as enabled by qualitative data analysis software. However, without testing the reliability of emerging unitizations, inferences drawn from them stand on epistemologically shaky ground.

I contend that the lack of concerns for the reliability of unitizing is due largely to the absence of suitable agreement measures. Guetzkow (1950) was the first to address the reliability of unitizing. Unfortunately, his coefficient measures the extent to which two observers disagree on the number of units they identify, not on whether units are of the same kind, leaving open the question of what the coders actually counted. Osgood's (1959, p. 44) and Holsti's (1969, p. 140) %-like index for whether two observers selected the same or different kinds of predefined units cannot recognize intersections of unequal units and, most importantly, fail to consider chance. I proposed a way to overcome these deficiencies (Krippendorff, 1995a, 2004) with a coefficient for unitizing that was to share the essential properties of the $\alpha$-family of coefficients. Unfortunately, its complexity has discouraged potential users. In the first printing of the 3rd edition of this book, I offered a simplification, but a French research group[1], running numerous tests with artificial reliability data, discovered an undesirable insensitivity that compels me to hereby withdraw this simplification.

This section presents two coefficients for segmenting or unitizing continua that have tested well. Section 12.4.2 replaces the defective agreement measure by a new ${}_U\alpha$-agreement coefficient. It assesses the degree to which units that were identified by any number of observers overlap in a given continuum and agree on their categorizations or valuations. It preserves the contiguity (integrity or coherence) of these units and ignores the irrelevant matter surrounding them. Section 12.4.3 presents a family of three ${}_u\alpha$-agreement coefficients. They extend ${}_c\alpha$ to partitioning a continuum into segments of various lengths and coding the relevant units among them. They afford analytical capabilities similar to ${}_c\alpha$ but at the expense of the contiguities within units. This is in line with the pursuit of research questions that call for quantifying volumes of textual and other unitized matter.

[1] Stéphane Bouvry, Yann Mathet and Antoine Widlöcher of the Université de Caen Basse-Normandie, Caen, France. Personal communication 2012.12.6

### 12.4.1 Reliability Data from Unitizing

Unitizing a continuum embraces three analytical operations. First, partitioning that continuum into mutually exclusive segments, second identifying among them the units of analysis, that are expected to answer given research questions, and third, categorizing or valuing these units. As required for any reliability test, data for assessing the reliability of unitizing must be generated by two or more independently working observers, judges, annotators, coders, or analysts and have sufficient diversity to be generalizable to the data whose reliability is in question. Figure 12.7 exemplifies such data schematically.

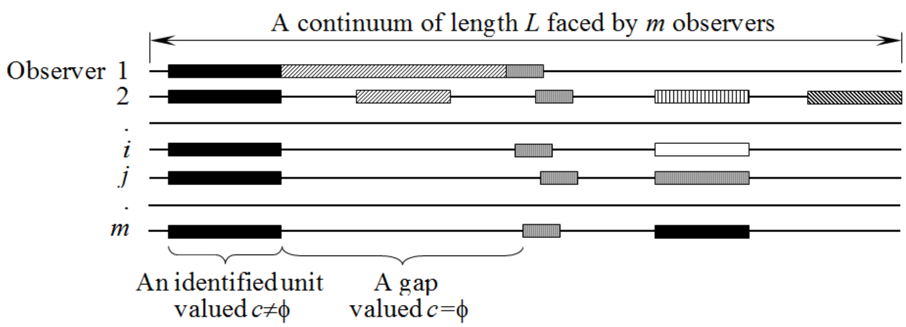


**Figure 12.7** Typical Reliability Data from Unitizing

Terms used in the following:

- Observers are labeled: 1, 2, …, $i$, $j$, …, $m \geq 2$.
- Each observer partitions a common continuum into segments $S$.
- Segments $S_{ig}$ and $S_{jh}$ are consecutively numbered:
  For observer $i$: 1, 2, 3, . . . , $g$, . . .
  For observer $j$: 1, 2, 3, . . . , $h$, . . .
- On the continuum, segments $S_{ig}$ are located by where they end: $End_{ig}$.
  Their beginning coincides with the end of their preceding segment $S_{ig\text{-}1}$: $End_{ig\text{-}1}$.
- Lengths are measured in integers
  Segments $S_{ig}$: $L(S_{ig})=End_{ig}-End_{ig\text{-}1}$
  Intersections $S_{ig}\cap S_{jh}\neq\{\}$: $L(S_{ig}\cap S_{jh})=\min(End_{ig}, End_{jh})-\max(End_{ig\text{-}1}, End_{jh\text{-}1})$
  Unions $S_{ig}\cup S_{jh}$: $L(S_{ig}\cup S_{jh})=\max(End_{ig}, End_{jh})-\min(End_{ig\text{-}1}, End_{jh\text{-}1})$
  The continuum: $L=\sum_g L(S_{ig})=\sum_h L(S_{jh})$
- Each segment is assigned a value $c$, $S_{ig \text{ valued c}}$, (or $k$):
  For identified units (relevant matter), $c$ is assigned by observers.
  For gaps between units (irrelevant matter), $c=\phi$ by default.
  When units are merely distinguished within a continuum, not variously valued, all identified units are valued identically $c\neq\phi$. (see Figure 12.8)

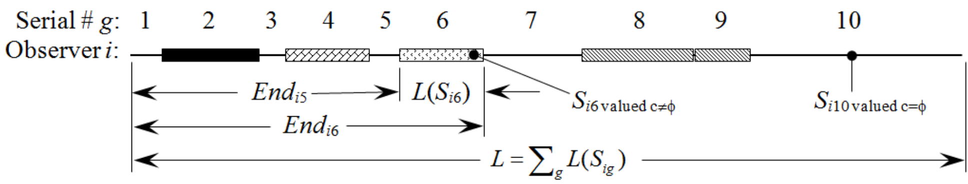


**Figure 12.8** Locating Segments - Units and Gaps between Them - on a Continuum

### 12.4.2 The $_U\alpha$-Agreement for Unitizing and Coding all Relevant Matter

For unitizing to be perfectly reliable, all units across all observers must occupy the same stretch on the continuum and be of the same kind. Deviations from either ideal are quantified by means of a difference function $_U\delta_{igjh}$. Figure 12.9 depicts three typical mismatches.

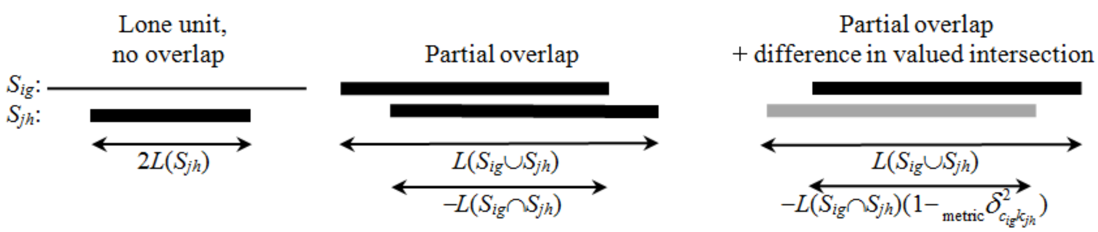


**Figure 12.9** Three Configurations of Differences between Identified Units

It shows the difference $_U\delta_{igjh}$ to be composed of two parts: the lengths of the non-intersecting parts of two units, evident in all three mismatiches in Figure 12.9 plus the lengths of their intersection, weighted by an applicable metric difference, found in the mismatch on the right. $_U\alpha$ accepts all metric differences $_{\text{metric}}\delta^2_{ck}$, defined in Section 12.3.3, plus one: When units are not categorized, valued, or regarded as such:

$$_{\text{metric}}\delta^2_{ck} = 0 =_{\text{no metric}}\delta^2_{ck} .$$

Accordingly, the difference $_U\delta_{igjh}$ between any two segments $S_{ig}$ and $S_{jh}$ is:

$$_U\delta_{igjh} = \begin{cases} L(S_{ig} \cup S_{jh}) - L(S_{ig} \cap S_{jh})(1 -_{\text{metric}}\delta^2_{c_{ig}k_{jh}}) & \text{iff } S_{ig} \cap S_{jh} \neq \{\} \text{ and } c_{ig} \neq \phi \neq k_{jh} \\ 2L(S_{ig}) & \text{iff } S_{ig} \subseteq S_{jh} \text{ and } c_{ig} \neq \phi \text{ and } k_{jh} = \phi \\ 2L(S_{jh}) & \text{iff } S_{jh} \subseteq S_{ig} \text{ and } c_{ig} = \phi \text{ and } k_{jh} \neq \phi \end{cases} \quad (16)$$

The observed disagreement $_UD_o$ is the average difference $_U\delta_{igjh}$ encountered in $m(m-1)/2$ pairs of continua unitized by $m$ observers:

$$_UD_o = \frac{\sum_i^{m-1}\sum_{j>i}^{m}\sum_{g,h}{}_U\delta_{igjh}}{N_o} \quad (17)$$

Where $N_o$ is the number of intersections that sum to the numerator of $_UD_o$.

The expected disagreement needs to randomize all pairs of units, ignoring references to their location on the continuum and the $m$ observers. To get a sense of the latter, consider the possible differences ${}_U\delta_{igjh}$ between two units of lengths $a$=4 and $b$=6 in Figure 12.10:

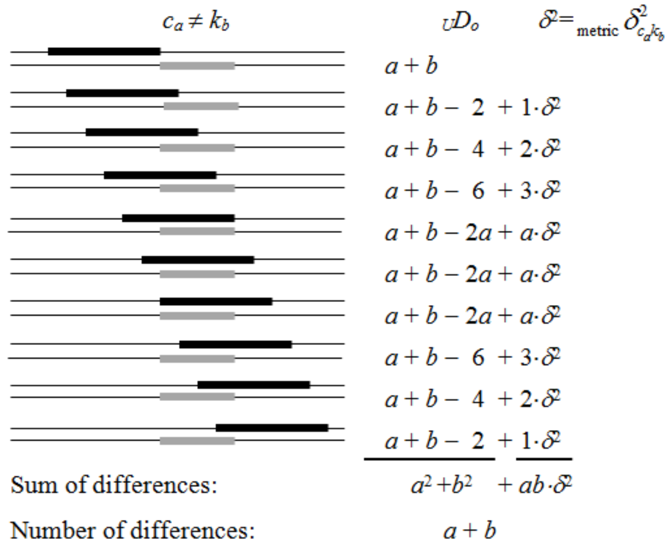


**Figure 12.10** Permutations of the Intersections of two Units: Sums and Numbers of Differences

It lists all possible intersections of these two units, their differences ${}_U\delta_{igjh}$, and provides their sums.

The expected disagreement ${}_UD_e$ needs to pair not two but all observers' units $S_{ig\ \text{valued}\neq\phi}$ with each other, except with themselves. Generalizing the proportion $\dfrac{a^2+b^2+ab\delta^2}{a+b}$ from Figure 12.10, the expected disagreement is defined as the average of all possible pairs of differences:

$$
{}_UD_e = \frac{\sum_i^m \sum_g \sum_j^m \sum_h \Big( \big(L(S_{ig\ \text{valued}\neq\phi})\big)^2 + \big(L(S_{jh\ \text{valued}\neq\phi})\big)^2 + L(S_{ig\ \text{valued}\neq\phi})L(S_{jh\ \text{valued}\neq\phi})\,{}_{\text{metric}}\delta^2_{c_{ig}k_{jh}} \Big) \text{ iff } S_{ig} \neq S_{jh}}{\sum_i^m \sum_g \sum_j^m \sum_h \big( L(S_{ig\ \text{valued}\neq\phi}) + L(S_{jh\ \text{valued}\neq\phi}) \big) \quad \text{iff } S_{ig} \neq S_{jh}} \qquad (18)
$$

This expression affords algebraic simplifications but is stated here for its resemblance to the results shown in Figure 12.10.

The ${}_U\alpha$-agreement for unitizing and categorizing or valuing relevant matter is:

$$
{}_U\alpha_{\text{metric}} = 1 - \frac{{}_UD_o}{{}_UD_e} \qquad (19)
$$

For a numerical example, consider the reliability of data in Figure 12.11:

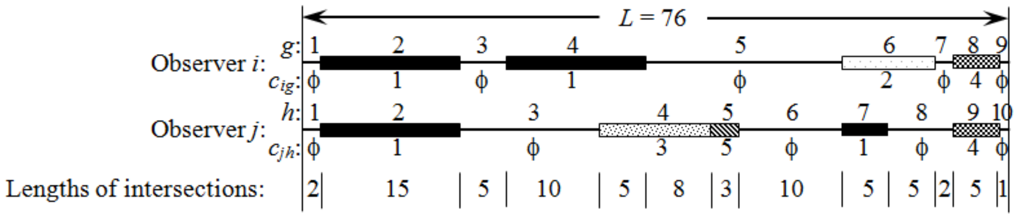


**Figure 12.11** Example of a Unitized Continuum

It depicts the segmentation of a continuum by two observers who collectively identified nine units, valued 1, 2, 3, 4 or 5, leaving ten gaps, labeled ϕ, between them unattended. There are $N_o$=5 intersections of units that contribute to the observed disagreement: two perfectly matching pairs, two partially overlapping ones, and one lone unit. ${}_U\alpha$ ignores the five intersections of matching irrelevant matter, the gaps between units.

When units are considered unordered or categorized and the nominal metric applies, according to (17), the observed disagreement becomes:

$$ {}_U D_o = \frac{(15-15(1-0)) + (23-5(1-1)) + 2\cdot 3 + (10-5(1-1)) + (5-5(1-0))}{5} = \frac{0+23+6+10+0}{5} = 7.8 $$

Calculating the expected disagreement By (18) requires more computational steps than can be shown here. It calls for summing 1376 differences from 9·(9-1)=72 pairs of units, yielding:

$$ {}_U D_\mathrm{e} = \frac{20966}{1376} = 15.2369 $$

Whereupon:

$$ {}_U \alpha_\text{nominal} = 1 - \frac{{}_U D_o}{{}_U D_e} = 1 - \frac{7.8}{15.2369} = 0.488 $$

The reliability that ignores all categorizations of units, which results from setting the difference function in ${}_U\delta_{igjh}$ in (16) to ${}_\text{no metric}\delta^2_{c_{ig}k_{jh}} = 0$, yields ${}_U\alpha_\text{no metric}$=0.515. Their difference is insignificant. If it were larger one could conclude that the categorization of units is less reliable than their mere identification. Both findings contrasts, however, with the reliability of valuing units on an interval scale from 1 to 5. Inserting the interval difference into (16) yields: ${}_U\alpha_\text{interval}$=0.616. This larger agreement is due to the presence of the smaller metric differences among neighboring values 1, 2, and 3, not involving units valued 4 and 5, which would lower the reliability for the interval coding of identified units.

When disagreement is absent, ${}_U\alpha$=1 and unitization can be considered perfectly reliable. When disagreement yields ${}_U\alpha$=0, it resembles chance. ${}_U\alpha$ can become negative when disagreement is systematic. Needless to say that the numerical example considered here would not be acceptable by any standard.

### 12.4.3 The ${}_u\alpha$-Agreements for Coding Segments of Unequal Lengths

The three coefficients of the ${}_u\alpha$-family are not limited to measure the agreement among identified units as is ${}_U\alpha$. Two of them evaluate the reliability of all segments, units and gaps, and one separately addresses the reliability of the valuation of units.

The family of ${}_u\alpha$ coefficients is defined by reference to a matrix of observed coincidences – just as is ${}_c\alpha$ – here, however, of the lengths of all intersections in a continuum, see Figure 12.12.

$$\ell_{ck} = \frac{1}{m-1}\sum_i^m \sum_{j\neq i}^m \sum_{g,h} L\left(S_{ig\ \text{valued}=c} \cap S_{jh\ \text{valued}=k}\right) \tag{20}$$

$$\begin{array}{c|cccccc|ll}
\text{Values:} & \phi & 1 & . & k & . & v & & \\
\hline
\phi & \ell_{\phi\phi} & \ell_{\phi 1} & . & \ell_{\phi k} & . & \ell_{\phi v} & \ell_{\phi\cdot} & \\
1 & \ell_{1\phi} & \ell_{11} & . & \ell_{1k} & . & \ell_{1v} & \ell_{1\cdot} & \ell_{1\cdot}^{*} = \ell_{1\cdot} - \ell_{1\phi} \\
. & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . \\
c & \ell_{c\phi} & \ell_{c1} & . & \ell_{ck} & . & \ell_{cv} & \ell_{c\cdot} = \sum_k \ell_{ck} & \ell_{c\cdot}^{*} = \ell_{c\cdot} - \ell_{c\phi} \\
. & . & . & . & . & . & . & . & . \\
v & \ell_{v\phi} & \ell_{v1} & . & \ell_{vk} & . & \ell_{vv} & \ell_{v\cdot} & \ell_{v\cdot}^{*} = \ell_{v\cdot} - \ell_{v\phi} \\
\hline
 & \ell_{\cdot\phi} & \ell_{\cdot 1} & . & \ell_{\cdot k} & . & \ell_{\cdot v} & \ell_{\cdot\cdot} = \sum_c \sum_k \ell_{ck} = mL & \ell_{\cdot\cdot}^{*} = \sum_{c\neq\phi} \sum_{k\neq\phi} \ell_{ck}
\end{array}$$

**Figure 12.12** The Matrix of Observed Coincidences of All Intersections

By analogy to (6) and (19), the observed disagreement of ${}_u\alpha$ is defined as:

$${}_uD_o = \frac{1}{\ell_{\cdot\cdot}} \sum_{c=\phi}^{v} \sum_{k=\phi}^{v} \ell_{ck}\ {}_{\text{metric}}\delta_{ck}^2 \tag{21}$$

Note that summing the lengths of intersecting segments in (20) effectively ignores their contiguities. In the following unitizations,

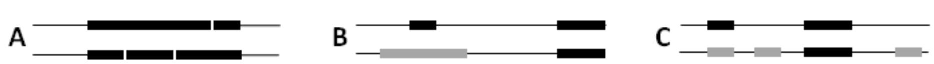


Figure 12.12 would represent A as in perfect agreement, ignore their non-matching distinctions, and could not distinguish between B and C. However, the contiguity of units is recognized in the definition of the expected coincidences:

$$\varepsilon_{ck} = \ell_{\cdot\cdot} \frac{\ell_{c\cdot}\ell_{\cdot k} - \left( \sum_i^m \sum_g \begin{Bmatrix} L\left(S_{ig\ \text{valued}\ c=\phi}\right) \\ \left(L\left(S_{ig\ \text{valued}\ c\neq\phi}\right)\right)^2 \end{Bmatrix} \text{iff } c=k \right)}{\ell_{\cdot\cdot}^2 - \sum_i^m \sum_g \begin{Bmatrix} L\left(S_{ig\ \text{valued}=\phi}\right) \\ \left(L\left(S_{ig\ \text{valued}\neq\phi}\right)\right)^2 \end{Bmatrix}} \tag{22}$$

The $_u\alpha$-agreement is defined in three ways: in general terms; in terms of observed and expected coincidences; and in terms of observed coincidences exclusively:

$$\begin{aligned} {}_u\alpha_{\text{metric}} &= 1 - \frac{{}_uD_o}{{}_uD_e} = 1 - \frac{\sum_{c=\phi}^{v}\sum_{k=\phi}^{v} \ell_{ck}\ {}_{\text{metric}}\delta_{ck}^2}{\sum_{c=\phi}^{v}\sum_{k=\phi}^{v} \varepsilon_{ck}\ {}_{\text{metric}}\delta_{ck}^2} = \\ &= 1 - \left(\ell_{\cdot\cdot} - \frac{1}{\ell_{\cdot\cdot}}\sum_i^m \sum_g \begin{cases} L(S_{ig\ \text{valued}=\phi}) \\ \left(L(S_{ig\ \text{valued}\neq\phi})\right)^2 \end{cases}\right) \frac{\sum_{c=\phi}^{v}\sum_{k=\phi}^{v} \ell_{ck}\ {}_{\text{metric}}\delta_{ck}^2}{\sum_{c=\phi}^{v} \ell_{c\cdot} \sum_{k=\phi}^{v} \ell_{\cdot k}\ {}_{\text{metric}}\delta_{ck}^2} \end{aligned} \tag{23a}$$

When the partition of the continuum includes blanks between units, labeled ϕ, only the nominal metric applies. This affords simplifications of (23a), also expressed in three ways:

$$\begin{aligned} {}_u\alpha_{\text{nominal}} &= 1 - \frac{{}_uD_o}{{}_uD_e} = 1 - \frac{\ell_{\cdot\cdot} - \sum_{c=\phi}^{v} \ell_{cc}}{\ell_{\cdot\cdot} - \sum_{c=\phi}^{v} \varepsilon_{ck}} = \\ &= 1 - \left(\ell_{\cdot\cdot} - \frac{1}{\ell_{\cdot\cdot}}\sum_i^m \sum_g \begin{cases} L(S_{ig\ \text{valued}=\phi}) \\ \left(L(S_{ig\ \text{valued}\neq\phi})\right)^2 \end{cases}\right) \frac{\ell_{\cdot\cdot} - \sum_{c=\phi}^{v} \ell_{cc}}{\ell^2_{\cdot\cdot} - \sum_{c=\phi}^{v} \ell^2_{c\cdot}} \end{aligned} \tag{23b}$$

Observe that the proportion on the right of (23a) for $_u\alpha_{\text{metric}}$ corresponds to the proportion in (8) for $_c\alpha_{\text{metric}}$. The seemingly complicated expression in its parenthesis deserves attention. Recall that the expected disagreement of $_c\alpha$ is obtained by pairing all values assigned to *n*.. units with each other but not with themselves. This amounts to $n_{\cdot\cdot}(n_{\cdot\cdot}-1)$ pairs, not $n^2_{\cdot\cdot}$ and explains the "$(n_{\cdot\cdot}-1)$" in definitions (5) and (8) of $_c\alpha$. The parenthesis in (23) accomplishes the same but for contiguous units of various lengths. If all segments were of equal length $L(S_{ig})=1$, this parenthesis would become $(\ell_{\cdot\cdot}-1)$, which proves $_u\alpha$ to be a generalization of $_c\alpha$ to contiguous units of unequal lengths. It may not be too obvious how the contiguity of units in $_u\alpha$ is preserved. It may be seen when (22) estimates the expected coincidences of matching values, $\varepsilon_{c=k}=\varepsilon_{cc}$. Here the squares $L(S_{ig\ \text{valued}=c\neq\phi})^2$ of each unit valued *c*, paired with itself, are subtracted from the square $\ell^2_{c\cdot}$ of the sum of all units valued *c*, leaving only units paired with each other in the numerator. The squaring does not apply to the lengths of gaps between units for lacking contiguity.

Accordingly, the example in Figure 12.11 would yield the following observed coincidences:

| Values: | $\phi$ | 1 | 2 | 3 | 4 | 5 | | |
|---|---|---|---|---|---|---|---|---|
| $\phi$ | 40 | 10 | 5 | 8 | . | 3 | $66=\ell_{\phi}.$ | |
| 1 | 10 | 30 | 5 | 5 | . | . | $50=\ell_{1}.$ | $40=\ell_{1}.^{*}$ |
| 2 | 5 | 5 | . | . | . | . | $10=\ell_{2}.$ | $5=\ell_{2}.^{*}$ |
| 3 | 8 | 5 | . | . | . | . | $13=\ell_{3}.$ | $5=\ell_{3}.^{*}$ |
| 4 | . | . | . | . | 10 | . | $10=\ell_{4}.$ | $10=\ell_{4}.^{*}$ |
| 5 | 3 | . | . | . | . | . | 3 | $0=\ell_{5}.^{*}$ |
| | 66 | 50 | 10 | 13 | 10 | 3 | $152=2{\cdot}76$ | $60=\ell..^{*}$ |

In these terms ${}_{u}\alpha$ measures:

$$ {}_{u}\alpha_{\text{nominal}} = 1-\left(152-\frac{\begin{matrix}15^2+15^2+10^2+5^2+31\\ +15^2+13^2+3^3+5^2+5^2+35\end{matrix}}{152}\right)\frac{152-40-30-10}{152^2-(66^2+50^2+10^2+13^2+10^2+3^3)} = $$

$$ = 1-(152-7.1974)\frac{72}{15870} = 0.343 $$

There are two descendants of ${}_{u}\alpha$. The first, ${}_{|u}\alpha$, assesses the reliability of the distinction between relevant and irrelevant matter, between identified units and the gaps between them. The second, ${}_{cu}\alpha$, evaluates what ${}_{|u}\alpha$ omits, the reliability of coding relevant matter.

${}_{|u}\alpha$ is obtained by applying (23) to what is in effect a 2-by-2 matrix of observed coincidences of $c{=}\phi$ versus $c{\neq}\phi$. Figure 12.12 marks its four quadrants by dotted lines. In this Figure's notations and taking advantage of simplifications in addition to (23b), ${}_{|u}\alpha$ becomes:

$$ {}_{|u}\alpha_{\text{binary}} = 1-\frac{{}_{|u}D_o}{{}_{|u}D_e} = 1-\left(\ell..-\frac{1}{\ell..}\sum_{i}^{m}\sum_{g}\left\{\begin{matrix}L\left(S_{ig\ \text{valued}=\phi}\right)\\ \left(L\left(S_{ig\ \text{valued}\neq\phi}\right)\right)^2\end{matrix}\right.\right)\frac{\ell_{\phi}.-\ell_{\phi\phi}}{\ell_{\phi}.(\ell..-\ell_{\phi}.)} \qquad (24) $$

In our example, (24) yields:

$$ {}_{|u}\alpha_{\text{binary}} = 1-\frac{0.3421}{0.5158} = 1-(152-7.197)\frac{66-40}{66(152-66)} = 0.337 $$

The other descendant of ${}_{u}\alpha$ answers the perhaps more common question concerning the reliability of coding unitized matter with various metrics. ${}_{cu}\alpha$ evaluates the $v$-by-$v$ sub-matrix of Figure 12.12 from which all coincidences with irrelevant matter, labeled $\phi$, are removed. This sub-matrix has its own marginal sums, listed on the right of Figure 12.12, marked by asterisks.

${}_{cu}\alpha$'s observed disagreement in this sub-matrix is:

$$ {}_{cu}D_o = \frac{\sum_{c=1}^{v}\sum_{k=1}^{v}\ell_{ck\ metric}\delta_{ck}^2}{\ell^{*}..} \qquad (25) $$

While the presence of unstructured matter in the continuum limits ${}_u\alpha$ to regard all values shown in Table 12.12 as nominal categories within which ${}_{|u}\alpha$ addresses only a binary distinction, ${}_{cu}\alpha$ accepts all applicable metrics, defined in Section 12.3.3.

Its expected coincidences in the same sub-matrix are defined by:

$$\varepsilon^*_{ck} = \frac{\ell^*_{c\cdot}\ell^*_{\cdot k} - \left(\frac{1}{m-1}\sum_i^m \sum_g \sum_{j\neq i}\left(\sum_h L\left(S_{ig\ \text{valued}c\neq\phi} \cap S_{jh\ \text{valued}k\neq\phi}\right)\right)^2 \text{ iff } c=k\right)}{\ell^*_{..} - \frac{1}{\ell^*_{..}(m-1)}\sum_i^m\sum_g\sum_{j\neq i}\left(\sum_h L\left(S_{ig\ \text{valued}\neq\phi}\cap S_{jh\ \text{valued}\neq\phi}\right)\right)^2} \quad (26)$$

It should be noted that (26) replaces definition (22) in the first printing of the 3rd edition of this book and everything that refers to it.

The data in Figure 12.11 yield the following two sub-matrices of contingencies. To illustrate how these coincidences are weighted, the matrix of the non-standardized interval differences ${}_{\text{interval}}\delta^2_{ck} = (c-k)^2$ is added to the two coincidences matrices.

Observed coincidences (see (20))

| Values: | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| 1 | 30 | 5 | 5 | . | 40 |
| 2 | 5 | . | . | . | 5 |
| 3 | 5 | . | . | . | 5 |
| 4 | . | . | . | 10 | 10 |
| | 40 | 5 | 5 | 10 | 60 |

Expected coincidences (see (26))

| | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| 1 | 22 | 4 | 4 | 8 | 38.0 |
| 2 | 4 | . | 0.5 | 1 | 5.5 |
| 3 | 4 | 0.5 | . | 1 | 5.5 |
| 4 | 8 | 1 | 1 | 1 | 11.0 |
| | 38 | 5.5 | 5.5 | 11 | 60.0 |

${}_{\text{interval}}\delta^2_{ck}$ (optional)

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | $1^2$ | $2^2$ | $3^2$ |
| 2 | $1^2$ | 0 | $1^2$ | $2^2$ |
| 3 | $2^2$ | $1^2$ | 0 | $1^2$ |
| 4 | $3^2$ | $2^2$ | $1^2$ | 0 |

With its expected disagreement analogous to (25):

$${}_{cu}D_e = \frac{\sum_{c=1}^{v}\sum_{k=1}^{v}\varepsilon_{ck}\ {}_{metric}\delta^2_{ck}}{\ell^*_{..}} \quad (27)$$

Entering (25) and (27) into the general form for α yields ${}_{cu}\alpha$:

$${}_{cu}\alpha_{metric} = 1 - \frac{{}_{cu}D_o}{{}_{cu}D_e} = 1 - \frac{\sum_{c=1}^{v}\sum_{k=1}^{v}\ell_{ck}\ {}_{\text{metric}}\delta^2_{ck}}{\sum_{c=1}^{v}\sum_{k=1}^{v}\varepsilon^*_{ck}\ {}_{\text{metric}}\delta^2_{ck}} \quad (28)$$

When the four values in this example are taken to constitute an interval scale:

$${}_{cu}\alpha_{\text{interval}} = 1 - \frac{2\left(5\cdot 1^2 + 5\cdot 2^2\right)}{2\left(4\cdot 1^2 + 4\cdot 2^2 + 8\cdot 3^2 + 0.5\cdot 1^2 + 1\cdot 2^2 + 1\cdot 1^2\right)} = 1 - \frac{50}{195} = 0.744$$

When the four values are considered unordered categories, it turns out that ${}_{cu}\alpha_{\text{nominal}}$=0.459 is significantly lower than ${}_{cu}\alpha_{\text{interval}}$=0.744. This difference is easily explained by pointing to the mismatching values in the off-diagonal cells of the two coincidence matrices. Evidently, the largest difference, here in the 4-1 and 1-4 cells have the highest expected coincidences but were not observed.

Note that both coincidence matrices of contain two empty cells in their diagonal.

This demonstrates the effect of not pairing unique units with themselves. How this is accomplished may be seen in the numerator of $\varepsilon^*_{c=k}$ in (26). Since the sub-matrix contains only one unit valued 2 and 3 respectively, there is no way for either kind to match by chance. Also, to avoid overestimating the expected agreement, the $\varepsilon^*_{11}$ and $\varepsilon^*_{44}$-cells of matching values need to exclude the effects of pairing units with themselves. For example, $\varepsilon^*_{44}$=1 results from subtracting $2{\cdot}5^2$ for pairing the two units, 5 in length, with themselves, from the square of their sum, $10^2$ / (60-10).

This correction also accounts for the seeming oddity that the marginal sums of the expected coincidences merely approximate those observed. However, marginal sums asymptotically approximate those observed when units are more numerous and relatively smaller, as would be expected in realistic data.

Finally, the example reveals the unequal coverage of these coefficients. ${}_u\alpha$ and ${}_{|u}\alpha$ account for 100% of the continuum, ${}_U\alpha$ for 56/76=74%, whereas ${}_{cu}\alpha$ accounts for only 60/152=29%, due to the intersections of valued units. These differences are due to how irrelevant matter is regarded. ${}_U\alpha$ accounts for all identified units, excluding only intersections among irrelevant matter. ${}_{cu}\alpha$ ignores all irrelevant matter, including its intersections with identified units. For researchers concerned with the reliability of valuing units, ${}_{cu}\alpha$'s unavoidable omissions are perfectly justified. The reliability of any kind of coding of intersections is adequately captured by ${}_{|u}\alpha$.

Free Java software to compute the above reliability coefficients for unitizing is available at https://mathet.users.greyc.fr/agreement/ (Accessed 2015.9.14), described in more details in (Krippendorff, Mathet, Bouvry, & Widlöcher, in press).